Reasoning by Analogical Proportion

S. Bayoudh

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1. Introduction

2. Analogy

3. Analogy on Sequences
   - Analogical Dissimilarity
   - Reasoning by Analogical Proportion
   - Application

4. Analogy on Binary and Nominal data
   - Classification Rule
   - Attributes Weighting
   - Results

5. Conclusion and Future Works
There are many kinds of Analogies:

- **Semantic Analogy**
  - cow : calf : : mare : foal
  - wings : bird : : fins : fish

- **Analogy on sequences (morphological)**
  - wolf : wolves : : leaf : leaves
  - unsafe : safely : : unfair : fairly

- **Analogy on numbers**
  - 1 : 2 : : 6 : 7
  - 4 : 2 : : 6 : 3
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Definition

a is to b as c is to d
Definition

\[ a \text{ is to } b \text{ as } c \text{ is to } d \]
\[ a : b :: c : d \]

captain : boat :: pilot : plane
Definition

a is to b as c is to d

a : b :: c : d

captain : boat :: pilot : plane

Symmetry of the relation "is to" :

c : d :: a : b

pilot : plane :: captain : boat

Means exchange :

a : c :: b : d

captain : plane :: pilot : boat
Definition

\[ a \text{ is to } b \text{ as } c \text{ is to } d \]
\[ a : b :: c : d \]

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Definition

A is to B as C is to D

a : b :: c : d

Captain : boat :: pilot : plane

Symmetry of the relation "is to" :

C : d :: a : b

Pilot : plane :: captain : boat

Means exchange :

A : c :: b : d

Captain : plane :: pilot : boat

C : a :: d : b

B : d :: a : c

C : d :: a : b

D : b :: c : a

A : c :: b : d

D : c :: b : a

A : b :: c : d

B : a :: d : c
With four elements, we have 24 different analogical equations but only three non equivalent

\[ a : b :: c : d \quad a : b :: d : c \quad a : d :: c : b \]

*Determinism*: si \( a : a :: b : x \) then \( x = b \)
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Let $\sum$ be an alphabet. $a,b,c,A,B,C$ the letters of the alphabet. We suppose that our alphabet is described with vectors. We introduce the new letter $-$ in the alphabet to represent the absence of a letter.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>!a</th>
<th>b</th>
<th>!b</th>
<th>c</th>
<th>!c</th>
<th>lowercase</th>
<th>capital</th>
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<tbody>
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<td>1</td>
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<td>B</td>
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<td>1</td>
</tr>
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Analogical Dissimilarity (AD) between letters

The AD is a measure that reflects how close are four objects from being in an analogical proportion. 

\[
AD(x, y, z, t) = \|x - y - z + t\|
\]

- \(AD(x, y, z, t) = 0\) ⇒ Analogical proportion
- \(AD(x, y, z, t) \uparrow\) ⇒ Analogical proportion \(\downarrow\)

For example:
- \(a : a : : b : b\) ⇒ \(AD(a, a, b, b) = 0\) ⇒ Analogical Proportion
- \(a : A : : b : B\) ⇒ \(AD(a, A, b, B) = 0\) ⇒ Analogical Proportion
- \(a : b : : A : c\) ⇒ \(AD(a, b, A, c) = 6\) ⇒ far from being in Analogy
The classical graphical representation of the analogical equation

\[ a : b :: c : d \]

is the parallelogram.
Analogical Dissimilarity between Sequences

Let Σ be an alphabet = {a, .., z, A, .., Z}
Let Σ* be the set of words or sequences = {U, V, ..}
Let the equation

\[ U : V :: W : T \]

\[ AD(U, V, W, T) = \sum_i AD(U_i, V_i, W_i, T_i) \]

<table>
<thead>
<tr>
<th>Unsafe</th>
<th>Safe</th>
<th>Fairly</th>
</tr>
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DA = 24

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DA = 0
Learning by Analogical Proportion

Let $E = (e_i)_{0 \leq i \leq \text{card}(E)} \subset \sum^*$ be the learning set. The learning step consists in finding the triplet $(e_i, e_j, e_k) \in E^3$ of words that forms an analogy with the element $x$ and which has the lowest AD.

$$e_i : e_j :: e_k : x \quad i, j, k \in [1, \text{card}(E)]$$

English
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$$e_i : e_j :: e_k : x \quad i, j, k \in [1, \text{card}(E)]$$
Resolution by Analogical Proportion

The Resolution by Analogical Proportion consists in creating the fourth element of the Analogy using the first three elements.

<table>
<thead>
<tr>
<th>J' Aime</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>-</th>
<th>le</th>
<th>tennis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Préfères tu le tennis?</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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J' aime - - - - - - le repos.
Préfères tu le repos?

J' aime le repos.
Préfères tu le tennis?

Français
```
Translation

Other application: how to pronounce English words (grapheme phoneme).
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Other application: how to pronounce English words (grapheme phoneme).
Application

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<tr>
<td>calf</td>
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<td>0</td>
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<tr>
<td>bull</td>
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\( \text{calf is to bull as kitten is to tomcat} \)
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\[
\text{calf is to bull as kitten is to tomcat}
\]

**Ruminant is to Ruminant as Feline is to ?**
### Analogy on Binary and Nominal data

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\[ \text{calf is to bull as kitten is to tomcat} \]

\[ \text{Ruminant is to Ruminant as Feline is to Feline} \]
Classification using the $k$ least dissimilar triplets

$$S = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\},$$

1. Keep those which involve solutions.
2. Computing the $AD$(triplet,$x$), triplet $\in S^3$.
3. Order triplets.
4. Deduce $k'$ from $k$.
5. Keep the $k'$ first triplets.
6. Find the winner class.

<table>
<thead>
<tr>
<th>$o_1 o_2 o_3$</th>
<th>$h(o_1) h(o_2) h(o_3)$</th>
<th>$AD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a b c$</td>
<td>$\omega_0 \omega_0 \omega_1$</td>
<td>$\omega_1$</td>
</tr>
<tr>
<td>$a b d$</td>
<td>$\omega_0 \omega_0 \omega_1$</td>
<td>$\omega_1$</td>
</tr>
<tr>
<td>$a b e$</td>
<td>$\omega_0 \omega_0 \omega_2$</td>
<td>$\omega_2$</td>
</tr>
<tr>
<td>$a c d$</td>
<td>$\omega_0 \omega_1 \omega_1$</td>
<td>$\perp$</td>
</tr>
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<td>$\vdots$</td>
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$$S = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x$$

1. Keep those which involve solutions.
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<td>$\omega_1$</td>
</tr>
<tr>
<td>$a b e$</td>
<td>$\omega_0 \omega_0 \omega_2$</td>
<td>$\omega_2$</td>
</tr>
<tr>
<td>$b a c$</td>
<td>$\omega_0 \omega_0 \omega_1$</td>
<td>$\omega_1$</td>
</tr>
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<td>$\omega_2$</td>
</tr>
<tr>
<td>$c a d$</td>
<td>$\omega_1 \omega_0 \omega_1$</td>
<td>$\omega_0$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
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</tr>
</tbody>
</table>
Classification using the $k$ least dissimilar triplets

\[ S = \{(c_i, h(c_i)) | 1 \leq i \leq m\}, x \]

1. Keep those which involve solutions.
2. Computing the AD(triplet, $x$), triplet $\in S^3$.
3. Order triplets.
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5. Keep the $k'$ first triplets.
6. Find the winner class.

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<thead>
<tr>
<th>$o_1 o_2 o_3$</th>
<th>$h(o_1) h(o_2) h(o_3)$</th>
<th>$AD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a b c$</td>
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</tbody>
</table>

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S. Bayoudh:
Reasoning by Analogical Proportion
Classification Rule

Classification using the $k$ least dissimilar triplets

$S = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x$

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<th>$AD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a b c$</td>
<td>$\omega_0 \omega_0 \omega_1 \omega_1$</td>
<td>6</td>
</tr>
<tr>
<td>$a b d$</td>
<td>$\omega_0 \omega_0 \omega_1 \omega_1$</td>
<td>1</td>
</tr>
<tr>
<td>$a b e$</td>
<td>$\omega_0 \omega_0 \omega_2 \omega_2$</td>
<td>3</td>
</tr>
<tr>
<td>$b a c$</td>
<td>$\omega_0 \omega_0 \omega_1 \omega_1$</td>
<td>4</td>
</tr>
<tr>
<td>$b a d$</td>
<td>$\omega_0 \omega_0 \omega_1 \omega_1$</td>
<td>0</td>
</tr>
<tr>
<td>$b a e$</td>
<td>$\omega_0 \omega_0 \omega_2 \omega_2$</td>
<td>3</td>
</tr>
<tr>
<td>$c a d$</td>
<td>$\omega_1 \omega_0 \omega_1 \omega_0$</td>
<td>7</td>
</tr>
</tbody>
</table>
Classification using the \( k \) least dissimilar triplets

\[ S = \{(c_i, h(c_i)) \mid 1 \leq i \leq m\}, x \]

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**Exemple**

<table>
<thead>
<tr>
<th>( o_1 o_2 o_3 )</th>
<th>( h(o_1) h(o_2) h(o_3) )</th>
<th>( h(x) )</th>
<th>( AD )</th>
</tr>
</thead>
<tbody>
<tr>
<td>b a d</td>
<td>( \omega_0 ) ( \omega_0 ) ( \omega_1 )</td>
<td>( \omega_1 )</td>
<td>0</td>
</tr>
<tr>
<td>c d e</td>
<td>( \omega_1 ) ( \omega_1 ) ( \omega_2 )</td>
<td>( \omega_2 )</td>
<td>1</td>
</tr>
<tr>
<td>a b d</td>
<td>( \omega_0 ) ( \omega_0 ) ( \omega_1 )</td>
<td>( \omega_1 )</td>
<td>1</td>
</tr>
<tr>
<td>d c e</td>
<td>( \omega_1 ) ( \omega_1 ) ( \omega_2 )</td>
<td>( \omega_2 )</td>
<td>2</td>
</tr>
<tr>
<td>d b c</td>
<td>( \omega_1 ) ( \omega_0 ) ( \omega_1 )</td>
<td>( \omega_0 )</td>
<td>2</td>
</tr>
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<td>a b e</td>
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<td>3</td>
</tr>
<tr>
<td>b a c</td>
<td>( \omega_0 ) ( \omega_0 ) ( \omega_1 )</td>
<td>( \omega_1 )</td>
<td>4</td>
</tr>
</tbody>
</table>
Classification Rule

Classification using the $k$ least dissimilar triplets

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<table>
<thead>
<tr>
<th>$o_1 o_2 o_3$</th>
<th>$h(o_1)\ h(o_2)\ h(o_3)$</th>
<th>$h(x)$</th>
<th>$AD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \ a \ d$</td>
<td>$\omega_0 \ \omega_0 \ \omega_1$</td>
<td>$\omega_1$</td>
<td>$0$</td>
</tr>
<tr>
<td>$c \ d \ e$</td>
<td>$\omega_1 \ \omega_1 \ \omega_2$</td>
<td>$\omega_2$</td>
<td>$1$</td>
</tr>
<tr>
<td>$a \ b \ d$</td>
<td>$\omega_0 \ \omega_0 \ \omega_1$</td>
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<td>$\omega_1 \ \omega_0 \ \omega_1$</td>
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<td>$2$</td>
</tr>
<tr>
<td>$a \ b \ e$</td>
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<td>$b \ a \ c$</td>
<td>$\omega_0 \ \omega_0 \ \omega_1$</td>
<td>$\omega_1$</td>
<td>$4$</td>
</tr>
</tbody>
</table>

### Table

<table>
<thead>
<tr>
<th>$k$</th>
<th>$1$</th>
<th>$2$</th>
<th>$3$</th>
<th>$4$</th>
<th>$5$</th>
<th>$6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k'$</td>
<td>$1$</td>
<td>$3$</td>
<td>$3$</td>
<td>$5$</td>
<td>$5$</td>
<td>$7$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h(x)$</th>
<th>$\omega_1$</th>
<th>$\omega_0$</th>
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<tbody>
<tr>
<td>$b\ a\ d$</td>
<td>$\omega_0\ \omega_0\ \omega_1$</td>
<td>$\omega_1$</td>
<td>0</td>
</tr>
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</tr>
<tr>
<td>$\vdots$</td>
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<td>$\vdots$</td>
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<td>$k'$</td>
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<td>5</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>$h(x)$</td>
<td>$\omega_1$</td>
<td>$\omega_1$</td>
<td>$\omega_1$</td>
<td>$?$</td>
<td>$?$</td>
<td>$\omega_2$</td>
</tr>
</tbody>
</table>
Fast search Algorithm : FADANA

Off line

- Preprocessing

On line

- Computation
- Elimination

\[ AD(u, v, z, t) \leq AD(z, t, y, x_i) - \delta \]
\[ AD(u, v, z, t) \geq AD(z, t, y, x_i) + \delta \]

- Selection

\[ (z, t) = \text{Argmin}_{(u, v) \in U} \sum_{(z, t) \in C} |AD(u, v, z, t) - AD(z, t, x_i, y)|\]
**Classification Rule**

**Fast search Algorithm : FADANA**

**Off line**
- Preprocessing

**On line**
- Computation
- Elimination

\[
AD(u, v, z, t) \leq AD(z, t, y, x_i) - \delta
\]

\[
AD(u, v, z, t) \geq AD(z, t, y, x_i) + \delta
\]

**Selection**

\[
(z, t^*) = \operatorname{Argmin}_{(u, v) \in U} \sum_{(z, t) \in C} |AD(u, v, z, t) - AD(z, t, x_i, y)|
\]

S. Bayoudh: Reasoning by Analogical Proportion
Fast search Algorithm : FADANA

Off line

- Preprocessing

On line

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AD(u, v, z, t) \leq AD(z, t, y, x_i) - \delta
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\[
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Fast search Algorithm : FADANA

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\[ AD(u, v, z, t) \leq AD(z, t, y, x_i) - \delta \]
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Selection

\[
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**Fast search Algorithm : FADANA**

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S. Bayoudh: Reasoning by Analogical Proportion
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Off line
- Preprocessing

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\[ \text{Selection} \]

\[ \delta \]

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Fast search Algorithm : FADANA

Off line
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AD(u, v, z, t) \geq AD(z, t, y, x_i) + \delta 
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Attributes Weighting

- Attributes.
- Departure class and arrival class.

Weighting Matrix \( W = (W_{kij})_{0 \leq k \leq d, 0 \leq i, j \leq C} \)
Attributes Weighting

- **Attributes.**
- **Departure class and arrival class.**

<table>
<thead>
<tr>
<th>$h(a)$</th>
<th>$h(b)$</th>
<th>$h(c)$</th>
<th>$h(x)$</th>
<th>resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0$</td>
<td>$\omega_0$</td>
<td>$\omega_0$</td>
<td>?</td>
<td>$h(x) = \omega_0$</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>$\omega_0$</td>
<td>$\omega_1$</td>
<td>?</td>
<td></td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>$\omega_1$</td>
<td>$\omega_0$</td>
<td>?</td>
<td></td>
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</tbody>
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Weighting Matrix $W = (W_{kij})_{0 \leq k \leq d, 0 \leq i,j \leq C}$
Attributes Weighting

- Attributes.
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<table>
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<td>$\omega_1$</td>
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<td>?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Weighting Matrix $W = (W_{kij})_{0 \leq k \leq d, 0 \leq i, j \leq C}$

<table>
<thead>
<tr>
<th>Departure Class</th>
<th>Arrived class (decision)</th>
</tr>
</thead>
<tbody>
<tr>
<td>class $\omega_i$</td>
<td>$W_{kii}$</td>
</tr>
<tr>
<td>class $\omega_j$</td>
<td>$W_{kji}$</td>
</tr>
</tbody>
</table>
\( W_{kij} \) = Estimating the probability of finding an analogy on attribute \( k \)
- \( \omega_i \) and \( \omega_j \) are departure and arrival classes.

1. Count the number of occurrences.
2. Estimate the probability.
Attributes Weighting

\[ W_{kij} = \text{Estimating the probability of finding an analogy on attribute } k \]

- \( \omega_i \) and \( \omega_j \) are departure and arrival classes.

1. **Count the number of occurrences.**

2. **Estimate the probability.**

\[
\begin{array}{ccc}
\ldots & \text{class } \omega_j & \ldots \\
\begin{array}{c}
\ldots \\
\ldots \\
\ldots \\
\end{array} & n_{0i} & \ldots \\
\begin{array}{c}
\ldots \\
a_k = 0 \\
a_k = 1 \\
\end{array} & n_{1i} & \ldots \\
\end{array}
\]

\[
\sum_{k=0}^{1} \sum_{i=1}^{C} n_{ki} = m
\]
Attributes Weighting

\(W_{kij} = \) Estimating the probability of finding an analogy on attribute \(k\)

- \(\omega_i\) and \(\omega_j\) are departure and arrival classes.

1. Count the number of occurrences.
2. Estimate the probability.

\[
P_k(1^{st}) = \frac{n_{0i}n_{0j}n_{0j}}{m^4}
\]

\[
\vdots
\]

\[
W_{kij} = P_k(1^{st}) + \cdots + P_k(6^{th})
\]

\[
W_{kij} = \frac{(n_{0i}^2 + n_{1i}^2)(n_{0j}^2 + n_{1j}^2) + 2 \cdot n_{0i}n_{0j}n_{1i}n_{1j}}{(6 \cdot m^4)}
\]
Attributes Weighting

\( W_{kij} \) = Estimating the probability of finding an analogy on attribute \( k \)

- \( \omega_i \) and \( \omega_j \) are departure and arrival classes.

1. Count the number of occurrences.
2. Estimate the probability.

Hence

\[
AD(a, b, c, x) = \sum_{k=1}^{d} W_{kij} AD(a_k, b_k, c_k, x_k)
\]
### Results

<table>
<thead>
<tr>
<th>Methods</th>
<th>MO.1</th>
<th>MO.2</th>
<th>MO.3</th>
<th>SP.</th>
<th>B.S</th>
<th>Br.</th>
<th>H.R</th>
<th>Mu.</th>
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</thead>
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<tr>
<td>nb nominal attributes</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>22</td>
<td>4</td>
<td>9</td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>nb binary attributes</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>22</td>
<td>4</td>
<td>9</td>
<td>4</td>
<td>22</td>
</tr>
<tr>
<td>nb instances train</td>
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<td>169</td>
<td>122</td>
<td>80</td>
<td>187</td>
<td>35</td>
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<tr>
<td>nb instances test</td>
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<td>432</td>
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<td>66</td>
<td>8043</td>
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<tr>
<td>nb classes</td>
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### Results

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**WAPC** \((k = 100)\)

**APC** \((k = 100)\)

**Decision Table**

**Id3**

**PART**

**Multi layer Perceptron**

**LMT**

**IB1**

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**S. Bayoudh:**

Reasoning by Analogical Proportion
1 Introduction

2 Analogy

3 Analogy on Sequences
   - Analogical Dissimilarity
   - Reasoning by Analogical Proportion
   - Application

4 Analogy on Binary and Nominal data
   - Classification Rule
   - Attributes Weighting
   - Results

5 Conclusion and Future Works
- Importance of the weighting.
- Apply it to numerical data.
- Improve the weighting.
- Reduce the computational time.
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Thanks for your attention.